DESIGNING SAMPLING PLAN ON SUSTAINABLE QUALITY REGION WITH PRODUCER'S PROTECTION

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ABSTRACT
This paper is to ascertain better quality for the lots of higher incoming quality with reasonable producer's risk. SQR and AQL indicate consumer's and producer's quality indices and the OC curve is restricted with high probability of acceptance at better quality levels. OC curves depicting Tightened Normal and Reduced quality of SQR is shown. Also examples were illustrated showing the practical use of the design at various production units.

Keywords:
Designing, sampling plan, design sampling plans, Sustainable planning

INTRODUCTION

Mostly sampling plans were designed and developed by pre-fixing proportional defectives like AQL, LQL, IQL, MAPD, tangent intercept, or the outgoing quality levels like AOQL and MAAOQ. Different authors like Peach and Littaur (1946), Cameron (1952), Norman Bush (1953), Mayer (1967), Soundarajan (1975), Govindaraju and Kuralmani (1992), Ramkumar(1996), were derived some basic operating procedure to locate sampling plan on the mentioned above indices. Also Ramkumar (2010) was first introduced the quality interval as new a quality measure. This paper is another initiative to improve the concept of quality interval known as Sustainable Quality Region (SQR)

The construction and designing of sampling plans in this paper was based on Sustainable Quality Region and producer's risk fixed at constant level (α=0.05) showing AQL. (Figure 1). Also it will be interested to engineers and technicians since SQR is a logical parameter based on AQL and MAPD approved by their desire. SQR is an interval and the producers were more preferred because they can execute the system more easily than a point quality index. The significance of SQR is upheld not only it is a range but also it is in terms of MAPD and AQL so that the acceptable probability is reasonably high upto $MAPD = AQL + SQR$ and it will be strictly declining beyond $MAPD = AQL + SQR$. Thus (AQL, SQR) design had high significance with respect to OC curve. So the producer can develop the required OC curve according to the demand of the product in terms of AQL and SQR. Generally SQR is expressible as multiple of AQL (say 1.5AQL, 0.8AQL, 2.38AQL etc) so that the quality controlling agencies and quality maintenance division of the production will get an idea of how much variability is permissible in the second parameter and where and how the inflection point of the OC curve is to be set. Thus fixing the OC curve will be easier in the beginning itself by selection of this sampling plan. As monotonic operating ratio do exist for AQL on SQR so that there will be a unique sampling plan for each of these combinations. For various values of SQR, the new sampling plans (n,c) were developed making sure that the accepted quality product has less cost of inspection and consumer's risk is reduced, fixing producer's risk at AQL.

DEFINITION OF SUSTAINABLE QUALITY REGION (SQR)

Is a range of a proportion of defectives between minimum quality- AQL assuring at least probability of acceptance 0.95 and maximum quality MAPD. Thus the interval of such quality will be $p^* - p_1$ is called the Sustainable Quality Region (SQR)

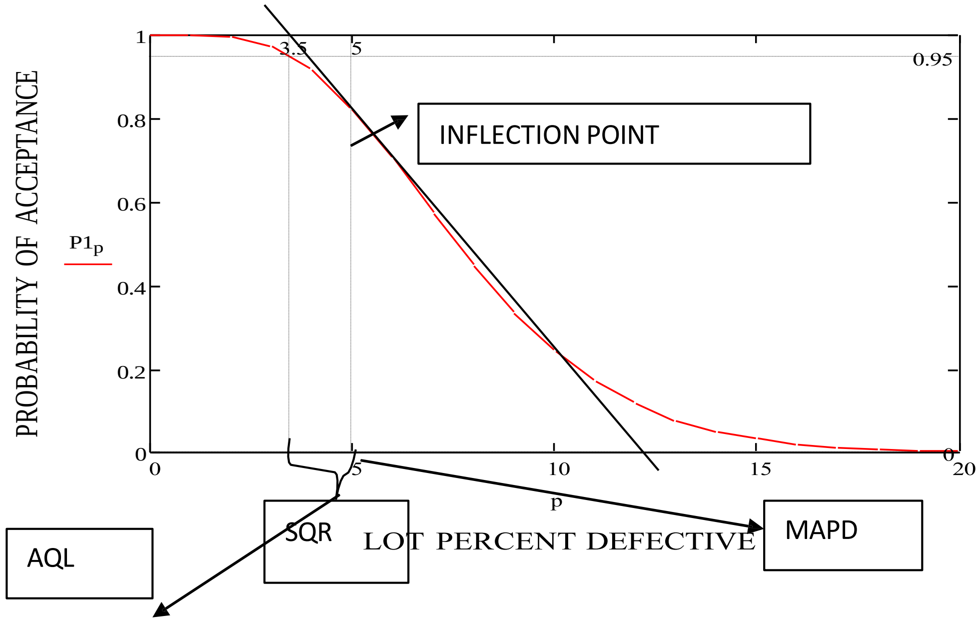


Figure 1: The OC curve which shows various quality measures used under this study

DESIGNING SAMPLING PLAN WITH SQR

Designing a sampling plan with SQR on producer's point of view refers to the quality level with the probability of acceptance of lot of specified defectives would be more accepted reducing producer's risk. Then, the plan for designing SSP with AQL is preferred so that producer's risk is fixed at 5% or 1% in general. The second quality level is fixed as SQR by which acceptability beyond AQL is controlled up to MAPD. Thus (AQL, SQR) is more producers friendly as well as protection are assured to consumers.

DETERMINATION OF SAMPLING PLAN

Fix AQL at 95% on and SQR with respect to MAPD and AQL in a production process. Construct an operating ratio

$$R = \frac{AQL}{SQR} = \frac{p_1}{p* - p_1} = \frac{np_1}{(np * - np_1)}$$

It is a monotonic increasing sequence of operating ratio corresponding to acceptance numbers (see Table:1). Using the Poisson unity values for AQL and SQR implies the existence of a monotone operating ratio and hence a unique sampling plan .Find appropriate table value of operating ratio R which is nearly less than or equal R= AQL/SQR and determine c from the table . Also find the value of np_1 or nSQR from the same table corresponding to selected c. Then $n = \frac{np_1}{p_1}$ Or $\frac{nSQR}{SQR}$ The values of np1, nSQR were given corresponding to c=1….40 in Table 1. The new design is efficient to contain the variability of quality that can be accommodated in terms of AQL. For example $SQR = 2 \times AQL$, or $SQR = 0.5 \times AQL$ will be a good measure for the producers to identify their quality of the product.

CONSTRUCTION OF THE PLAN

The number of defectives in large production is assumed to be very small and the probability of defective is less that 0.10 so that the distribution of the number of defect or defectives in a lot of size N (large) follows Poisson distribution. Let a sample of n is inspected with probability of defective in lot p and c is acceptance number, then the probability of acceptance of the lot with c defectives is

$$P_a(p) = \sum_{r=0}^{c} \frac{e^{-np}(np)^r}{r!} \quad \ldots\ldots 1$$

Therefore, the values of np_1 at 5% level will be obtained from the following inequality given below

$$P_a(p_1) = \sum_{r=0}^{c} \frac{e^{-np_1}(np_1)^r}{r!} \geq 0.95 \quad \text{.......} 2$$

Also by definition, the point of inflection of a continuous function Pa(p) is obtained at

$$P_a''(p) = 0$$

Solving the above equation $p^* = \frac{c}{n}$. so that np*=c 3

Then the operating ratio $\frac{np1}{np^*-np_1} = \frac{p_1}{(p^*-p_1)} = \frac{AQL}{SQR}$ = R is obtained and hence c and n is determined.

CONSTRUCTION OF TABLES

Values of np_1, and np* were obtained using equation, 2 and 3, hence Table 1 was developed to shows the values of c, R, $nSQR$, $nAQL$ for c=1,2...40 where $nSQR = (np^* - np_1)$, $c = np^*$ and $nAQL = np_1$. Table 2, represents some sampling plans corresponding to specified AQL and SQR. The operating ratio for each pairs of (AQL, SQR) is calculated and corresponding sampling plans were developed . Table 3 shows (AQL, SQR) for various combination of (n,c). It was constructed by finding $nSQR$ and $nAQL$ from table 1 and hence AQL and SQR for the specified values of n. Table 4 is a conversion table to identify other quality indices of the designed plan like, MAPD, AOQL and MAAOQ. The values of $n\,MAPD, nAOQL$ and $nMAAOQ$, were also developed.

EXAMPLE:1

For a computer component AQL =1.5% and SQR = 2 times AQL. The quality indices are p_1=.015 and $(p^* - p_1) = 2 * .015 = 0.03$, Then $R = \frac{0.03}{0.015} = 2$, From Table 1 approximate $R = 2.0211$ (exceeding $R = 1.945$), the corresponding $c = 15$

Then $n = \frac{np_1}{p_1} = \frac{10.035}{0.015} = 669$. The needed sampling plan to test the quality of the computer component is $(669,15)$. Then Using Table 3, since c =np* =15 so $p^* = \frac{19}{669} = 0.0224$, also $nAOQL = 10.134$ so $AOQL = \frac{10.134}{669} = 0.015$ $nMAAOQ = 8.521$

so $MAAOQ = \frac{8.521}{669} = 0.0127$.

EXAMPLE:2

A house hold article (Plastic bucket) is designed with $AQL = 3.\ \%$ defective and $SQR = 3.5\%$ defectives, then from Table 2, the required sampling plan is (46,3)

EXAMPLE:3

For a product manufactures accepted an OC curve and to keep the quality, they are in need of AQL and SQR . What is AQL and SQR for a sampling plan (50,2), From table:3 the value of (AQL,SQR) is (1.634, 2.36)

COMPARISON OF OC CURVES

Suppose the SQR is defined as multiple of AQL say 1/2 times, 1 times and 2 times. For example AQL= 0.03, then SQR=0.015, 0.03, 0.06 respectively. Using Table 1, the sampling plans were $(317,14)$, $(68,4)$ and $(11,1)$ respectively. When SQR is 2 times AQL, OC curve is so liberal containing large percentage of defectives in the accepted lot (refer $P_3a(p)$). From the figure about 37% defects are accepted as LTPD and good lots were rejected. But when SQR=1xAQL, the defectives in the accepted lot is controlled and LTPD is 12.5% ($P_2a(p)$) . Decreasing the multiple relation to ½ a very stringent OC curve is formulated with LTPD=8%.($P_1a(p)$). Thus for very high quality production, it is advisable to use fractional multiple of AQL as SQR, while in moderate quality, a multiple nearby 1 is advisable. If the product is liberally produced the multiplicative SQR to AQL in a range (1.5—2.5) can be adopted.

Table 1: Operating ratio R

c	R	np1	nSQR	c	R	np1	nSQR	c	R	np1	nSQR
1	0.5503	0.355	0.645	15	2.0211	10.035	4.965	29	2.9157	21.594	7.406
2	0.6920	0.818	1.182	16	2.0953	10.831	5.169	30	2.9703	22.444	7.556
3	0.8359	1.366	1.634	17	2.1675	11.633	5.367	31	3.0249	23.298	7.702
4	0.9704	1.97	2.03	18	2.2385	12.442	5.558	32	3.0774	24.152	7.848
5	1.0946	2.613	2.387	19	2.3066	13.254	5.746	33	3.1301	25.01	7.99
6	1.2107	3.286	2.714	20	2.3738	14.072	5.928	34	3.1820	25.87	8.13
7	1.3186	3.981	3.019	21	2.4392	14.894	6.106	35	3.2326	26.731	8.269
8	1.4205	4.695	3.305	22	2.5026	15.719	6.281	36	3.2826	27.594	8.406
9	1.5181	5.426	3.574	23	2.5647	16.548	6.452	37	3.3325	28.46	8.54
10	1.6102	6.169	3.831	24	2.6264	17.382	6.618	38	3.3814	29.327	8.673
11	1.6987	6.924	4.076	25	2.6862	18.218	6.782	39	3.4298	30.196	8.804
12	1.7842	7.69	4.31	26	2.7453	19.058	6.942	40	3.4772	31.066	8.934
13	1.8659	8.464	4.536	27	2.8028	19.9	7.1				
14	1.9448	9.246	4.754	28	2.8599	20.746	7.254				

Table 2: Sampling plan for specified AQL and SQR.

AQL	SQR						
	0.015	0.02	0.025	0.03	0.035	0.04	0.045
0.01	(82,2)	(36,1)	(36,1)	(36,1)	(36,1)	(36,1)	(36,1)
0.02	(199,7)	(99,4)	(68,3)	(41,2)	(18,1)	(18,1)	(18,1)
0.03	(308,14)	(181,9)	(109,6)	(66,4)	(46,3)	(46,3)	(27,2)
0.04	(455,25)	(231,14)	(154,10)	(100,7)	(65,5)	(49,4)	(34,3)
0.05	(569,37)	(314,22)	(185,14)	(138,11)	(94,8)	(80,7)	(52,5)
0.06		(374,30)	(234,20)	(154,14)	(115,11)	(90,9)	(66,7)
0.07		(444,40)	(284,27)	(189,19)	(132,14)	(110,12)	(88,10)
0.08			(324,34)	(228,25)	(166,19)	(116,14)	(96,12)
0.09				(249,30)	(184,23)	(147,19)	(103,14)
0.1				(285,37)	(199,27)	(157,22)	(116,17)

Sampling plan for AQL and SQR (continued)

AQL	SQR						
	0.05	0.055	0.06	0.065	0.07	0.075	0.08
0.01	(36,1)	(36,1)	(36,1)	(36,1)	(36,1)	(36,1)	(36,1)
0.02	(18,1)	(18,1)	(18,1)	(18,1)	(18,1)	(18,1)	(18,1)
0.03	(12,1)	(12,1)	(12,1)	(12,1)	(12,1)	(12,1)	(12,1)
0.04	(34,3)	(20,2)	(20,2)	(9,1)	(9,1)	(9,1)	(9,1)
0.05	(39,4)	(39,4)	(27,3)	(27,3)	(16,2)	(16,2)	(7,1)
0.06	(55,6)	(44,5)	(33,4)	(33,4)	(23,3)	(23,3)	(23,3)
0.07	(69,8)	(57,7)	(37,5)	(37,5)	(28,4)	(28,4)	(28,4)
0.08	(77,10)	(68,9)	(50,7)	(41,6)	(33,5)	(33,5)	(25,4)
0.09	(88,12)	(69,10)	(60,9)	(52,8)	(44,7)	(37,6)	(29,5)
0.1	(92,14)	(77,12)	(77,12)	(54,9)	(47,8)	(40,7)	(40,7)

Table 3: AQL and SQR for various combinations of (n,c) (Max: AQL=20%).

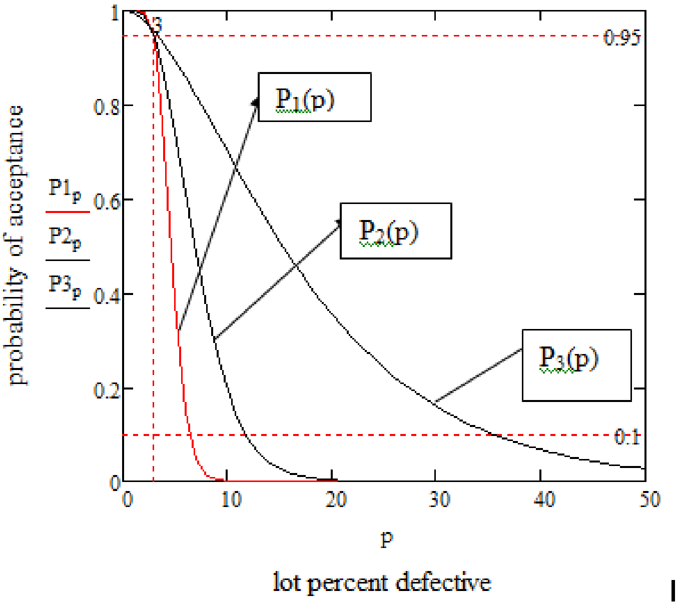
c	N											
	10		25		50		100		200		500	
	AQL	SQR	AQL	SQR	AQL	SQR	AQL	SQR	AQL	SQR	AQL	SQR
1	0.0355	0.0645	0.0142	0.0258	0.0071	0.0129	0.0035	0.0064	0.0017	0.0032	0.0007	0.0012
2	0.0818	0.1182	0.0327	0.0472	0.0163	0.0236	0.0081	0.0118	0.004	0.0059	0.0016	0.0023
3	0.1366	0.1634	0.0546	0.0653	0.0273	0.0326	0.0136	0.0163	0.0068	0.0081	0.0027	0.0032
4			0.0788	0.0812	0.0394	0.0406	0.0197	0.0203	0.0098	0.0101	0.0039	0.004
5			0.1045	0.0954	0.0522	0.0477	0.0261	0.0238	0.013	0.0119	0.0052	0.0047
6			0.1314	0.1085	0.0657	0.0542	0.0328	0.0271	0.0164	0.0135	0.0065	0.0054
7			0.1592	0.1207	0.0796	0.0603	0.0398	0.0301	0.0199	0.015	0.0079	0.006
8			0.1878	0.1322	0.0939	0.0661	0.0469	0.033	0.0234	0.0165	0.0093	0.0066
9					0.1085	0.0714	0.0542	0.0357	0.0271	0.0178	0.0108	0.0071
10					0.1233	0.0766	0.0616	0.0383	0.0308	0.0191	0.0123	0.0076
11					0.1384	0.0815	0.0692	0.0407	0.0346	0.0203	0.0138	0.0081
12					0.1538	0.0862	0.0769	0.0431	0.0384	0.0215	0.0153	0.0086
13					0.1692	0.0907	0.0846	0.0453	0.0423	0.0226	0.0169	0.009
14					0.1849	0.095	0.0924	0.0475	0.0462	0.0237	0.0184	0.0095
15					0.2007	0.0993	0.1003	0.0496	0.0501	0.0248	0.02	0.0099
16							0.1083	0.0516	0.0541	0.0258	0.0216	0.0103
17							0.1163	0.0536	0.0581	0.0268	0.0232	0.0107
18							0.1244	0.0555	0.0622	0.0277	0.0248	0.0111
19							0.1325	0.0574	0.0662	0.0287	0.0265	0.0114
20							0.1407	0.0592	0.0703	0.0296	0.0281	0.0118

Table 4: Conversion table for AQL and SQR

c	n AQL	n SQR	R	n A OQL	nMA AOQ	c	n AQL	n SQR	R	n A OQL	n MA AOQ
1	0.355	0.645	0.5503	0.84	0.736	21	14.894	6.106	2.4392	14.6569	11.711
2	0.818	1.182	0.6920	1.371	1.353	22	15.719	6.281	2.5026	15.4269	12.24
3	1.366	1.634	0.8359	1.9419	1.942	23	16.548	6.452	2.5647	16.2000	12.768
4	1.97	2.03	0.9704	2.544	2.515	24	17.382	6.618	2.6264	16.9759	13.296
5	2.613	2.387	1.0946	3.168	3.08	25	18.218	6.782	2.6862	17.756	13.823
6	3.286	2.714	1.2107	3.8118	3.638	26	19.058	6.942	2.7453	18.5400	14.35
7	3.981	3.019	1.3186	4.4719	4.191	27	19.9	7.1	2.8028	19.3260	14.875
8	4.695	3.305	1.4205	5.146	4.74	28	20.746	7.254	2.8599	20.1150	15.401
9	5.426	3.574	1.5181	5.8310	5.287	29	21.594	7.406	2.9157	20.9069	15.926
10	6.169	3.831	1.6102	6.528	5.83	30	22.444	7.556	2.9703	21.702	16.451
11	6.924	4.076	1.6987	7.2329	6.372	31	23.298	7.702	3.0249	22.4989	16.975
12	7.69	4.31	1.7842	7.9479	6.912	32	24.152	7.848	3.0774	23.2979	17.499
13	8.464	4.536	1.8659	8.6699	7.45	33	25.01	7.99	3.1301	24.0999	18.022
14	9.246	4.754	1.9448	9.3980	7.986	34	25.87	8.13	3.1820	24.9040	18.545
15	10.035	4.965	2.0211	10.134	8.521	35	26.731	8.269	3.2326	25.711	19.063
16	10.831	5.169	2.0953	10.8750	9.055	36	27.594	8.406	3.2826	26.5190	19.59
17	11.633	5.367	2.1675	11.6219	9.588	37	28.46	8.54	3.3325	27.3300	20.112
18	12.442	5.558	2.2385	12.3739	10.12	38	29.327	8.673	3.3814	28.1420	20.634
19	13.254	5.746	2.3066	13.1309	10.65	39	30.196	8.804	3.4298	28.9560	21.155
20	14.072	5.928	2.3738	13.892	11.18	40	31.066	8.934	3.4772	29.773	21.677

Figure 2: Multiplicative property for specified SQR when AQL is fixed

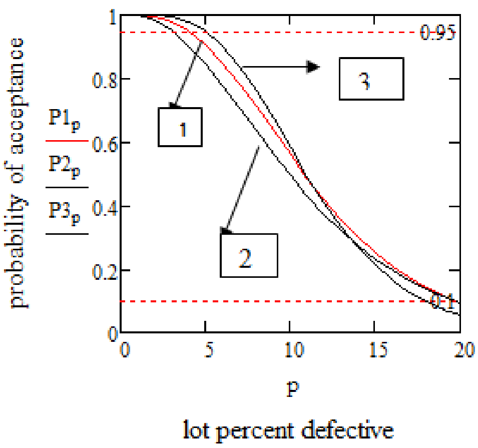
gure 2: Multiplicative property for specified SQR when AQL is fixed



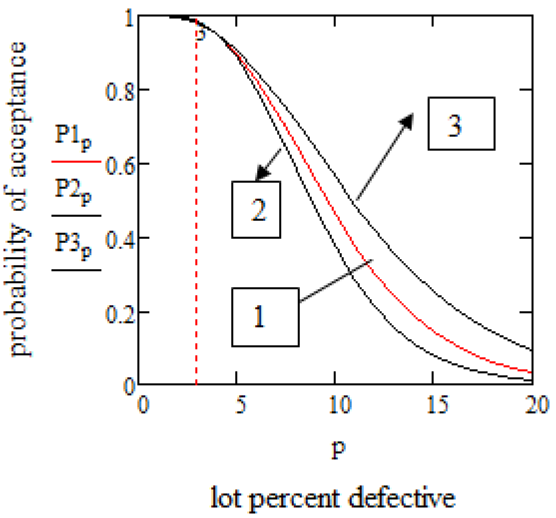
Where

1. $P_1(p)$ represents the probability of acceptance for SSP of (317,14)
2. $P_2(p)$ represents the probability of acceptance for SSP of (68,4)
3. $P_3(p)$ represents the probability of acceptance for SSP of (11,1)

F igure 3: The power of acceptance in the OC curve at various AQL and SQR= 4.5%. When AQL = 4% (Normal), then, SSP was (34,3), when AQL = 3%(Tightened), SSP was (27,2) and when AQL=5%(Relaxed), SSP was (52,5)



F igure 4: The power of acceptance in the OC curve at various SQR and AQL= 3%. When SQR = 4% (Normal), then, SSP was (49,4), when SQR = 3.5% (Tightened), SSP was (65,5) and when SQR=4.5%(Reduced), SSP was (34,3)



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